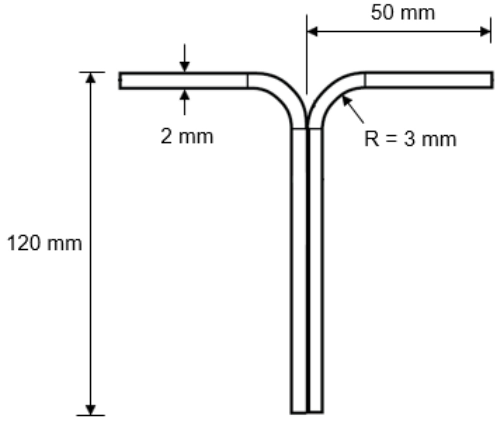
EC3 1-3 2006 CFFD Example 005

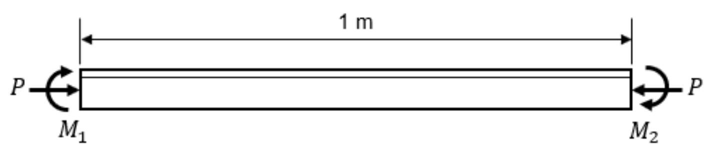
T-SECTION MEMBER UNDER COMBINED COMPRESSION, BENDING, AND SHEAR

EXAMPLE DESCRIPTION

Compression, moment, and shear capacities and demand/capacity ratio are calculated for T section at mid-span as shown below. It is simply supported with a length of 1.0 meter.

GEOMETRY, PROPERTIES AND LOADING





Dead: $P = 1000\ N, M_1 = 3{,}000{,}000\ N - mm, M_2 = 500{,}000\ N - mm$

TECHNICAL FEATURES TESTED

- Axial compressive strength
- Major moment strength
- Shear strength
- Demand/Capacity ratio.

COMPUTER FILE: EC3 1-3 2006 CFFD EX005

Applicable Programs

- SAP2000

RESULTS COMPARISON

Independent results are hand calculated.

CONCLUSION

The results show exact match with independent results.

Benchmarks: SAP2000

Output Parameter	Program	Independent	Percent Difference
Axial - Flexural buckling $N_{b,Rd}\ (N)$	91838	91853	0.02%
Axial – Torsional-Flexural buckling $N_{b,Rd}\ (N)$	16096	16092	0.02%
Axial – Local & Distortional Buckling $N_{c,Rd}\ (N)$	114939	114940	0.00%
Flexure – Lateral-Torsional Buckling $M_{b,Rd}\ (N-mm)$	5728005	5728005	0.00%
Flexure – Local & Distortional Buckling $M_{c,Rd}\ (N-mm)$	5958202	5958190	0.00%
Shear $V_{b,Rd}\ (N)$	96628	96628	0.00%
D/C Ratio	0.704	0.704	0.00%

HAND CALCULATION

Properties:

Material: $E = 210{,}000\ N/mm^2$, $G = 80{,}770\ N/mm^2$, $f_{yb} = 350\ N/mm^2$

Section: $h = 120\ mm$, $b = 50\ mm$, $t = 2\ mm$, $r = 3\ mm$

$\rightarrow h_p = h - t/2 = 120 - 2/2 = 119\ mm$

$\rightarrow b_p = b - t/2 = 50 - 2/2 = 49\ mm$

Check for the effect of rounding of the corners:

$$\frac{r}{t} = \frac{3}{2} = 1.5 < 5 \rightarrow OK$$

$$\frac{r}{b_p} = \frac{3}{49} = 0.06 < 0.1 \rightarrow OK$$

Therefore, the effect of rounding of the corners can be neglected in calculation of section properties:

$A_g = 672\ (mm^2)$

$I_y = 1053224.4\ (mm^4)$

$I_z = 166649.6\ (mm^4)$

$i_y = 39.6\ (mm)$

$i_z = 15.75\ (mm)$

$W_{el,c} = 24990\ (mm^3)$

$W_{el,t} = 13704.2\ (mm^3)$

$I_t = 896\ (mm^4)$

$I_w = 0.0\ (mm^6)$

$y_0 = 0.0\ (mm)$

$z_0 = 42.15\ (mm)$

Member: $K_y = K_z = K_T = 1.0$ for a pinned-pinned condition

$L_y = L_z = L_T = 1000\ mm$

$k_{yy} = k_{zz} = k_{yz} = k_{zy} = 1.0$

Loadings: Dead: $P = 1000\ N, M_1 = 3{,}000{,}000\ N - mm, M_2 = 500{,}000\ N - mm$

Required strengths: for the section in the middle

$N_{Ed} = 1.0D = 1.0 \times 1000 = 1000\ (N)$

$M_{Ed} = 1.0D = 1250000\ (N - mm)$

$V_{Ed} = 1.0D = 3500\ (N)$

Member Compression Capacity: the compression capacity is calculated considering the limit states of global buckling, and local buckling. Distortional buckling is not considered as there is no lip stiffener.

1. Local buckling:

   The effective width method is utilized to calculate the nominal axial strength in consideration of local buckling with the compressive stress of $f_{yb} = 350\ (N/mm^2)$.

   Check for the applicability of the method as the following conditions are satisfied:

   $$\frac{b}{t} = \frac{50}{2} = 25 < 50 \rightarrow OK$$
   $$\frac{h}{2t} = \frac{120}{2 \times 2} = 30 < 500 \rightarrow OK$$

   As the section is subjected to uniform compression and both flanges are considered outstand unstiffened elements:

   $$\psi = 1$$
   $$k_\sigma = 0.43$$
   $$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194$$
   $$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{49/2}{28.4 \times 0.8194\sqrt{0.43}} = 1.6055 > 0.748$$
   $$\rho = \frac{\bar{\lambda}_{p,b} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{1.6055 - 0.188}{1.6055^2} = 0.55 \le 1.0$$
   $$b_{eff} = \rho b_p = 0.55 \times 49 = 26.95\ (mm)$$

   The web is considered an outstand unstiffened element under uniform compression:

   $$\psi = 1$$
   $$k_\sigma = 0.43$$
   $$\bar{\lambda}_{p,b} = \frac{h_p/(2t)}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{119/4}{28.4 \times 0.8194\sqrt{0.43}} = 1.95 > 0.748$$
   $$\rho = \frac{\bar{\lambda}_{p,b} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{1.95 - 0.188}{1.95^2} = 0.463 \le 1.0$$
   $$h_{eff} = \rho h_p = 0.463 \times 119 = 55.15\ (mm)$$
   $$A_{eff} = 2th_{eff} + 2tb_{eff} = 2 \times 2 \times 55.15 + 2 \times 2 \times 26.95 = 328.4\ (mm^2)$$
   $$A_{eff} = 328.4\ (mm^2) < 672\ (mm^2) = A_g$$
   $$\rightarrow N_{c,Rd} = \frac{A_{eff} f_{yb}}{\gamma_{M0}} = \frac{328.4 \times 350}{1.0} = 114940\ (N)$$

   Because the section is symmetric about z-z axis, its effective properties are also symmetric about z-z axis, resulting in $e_{Nz} = 0 \rightarrow \Delta M_{z,Ed} = 0$

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = \frac{2th_p \frac{h_p}{2}}{A} = \frac{2 \times 2 \times 119 \frac{119}{2}}{672} = 42.146\ (mm)$$

$$\bar{y}_{eff} = \frac{\sum_i A_{eff,i} z_i}{A_{eff}} = \frac{2th_{eff} \frac{h_{eff}}{2}}{A_{eff}} = \frac{2 \times 2 \times 55.15 \times \frac{55.15}{2}}{328.4} = 18.523\ (mm)$$

$$e_{Ny} = \bar{z}_{eff} - \bar{z} = 18.523 - 42.146 = -23.623\ (mm)$$

$$\Delta M_{y,Ed} = N_{Ed} e_{Ny} = 1200 \times (-23.613) = -28347.6\ (N - mm)$$

Since the moment demand at mid-span is positive but $\Delta M_{y,Ed}$ is negative and results in favourable design, therefore $\Delta M_{y,Ed}$ is taken to be zero for a more conservative design.

$$\Delta M_{y,Ed} = 0.0\ (N - mm)$$

2. Global buckling: includes flexural buckling and torsional and flexural-torsional buckling
   i. Flexural buckling:

$$N_{cr,y} = \frac{\pi^2 E I_y}{\left(K_y L_y\right)^2} = \frac{\pi^2 (210{,}000) 1053224.4}{(1.0 \times 1000)^2} = 2182930.7\ (N)$$

$$N_{cr,z} = \frac{\pi^2 E I_z}{(K_z L_z)^2} = \frac{\pi^2 (210{,}000) 166649.6}{(1.0 \times 1000)^2} = 345400.8\ (N)$$

$$\bar{\lambda}_y = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,y2}}} = \sqrt{\frac{328.4 \times 350}{2182930.72}} = 0.229$$

$$\bar{\lambda}_z = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,z2}}} = \sqrt{\frac{328.4 \times 350}{345400.8}} = 0.577$$

For T section with lips, the buckling curve is c and $\alpha = 0.49$

$$\Phi_y = 0.5\left[1 + \alpha\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^2\right] = 0.5[1 + 0.49(0.229 - 0.2) + 0.229^2] = 0.534$$

$$\Phi_z = 0.5\left[1 + \alpha\left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^2\right] = 0.5[1 + 0.49(0.577 - 0.2) + 0.577^2] = 0.759$$

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y^2}} = \frac{1}{0.534 + \sqrt{0.534^2 - 0.229^2}} = 0.985$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}} = \frac{1}{0.759 + \sqrt{0.759^2 - 0.577^2}} = 0.799$$

$$N_{by,Rd} = \frac{\chi_y A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.985 \times 328.4 \times 350}{1.0} = 113216\ (N)$$

$$N_{bz,Rd} = \frac{\chi_z A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.799 \times 328.4 \times 350}{1.0} = 91853.4\ (N)$$

   ii. Torsional and flexural-torsional buckling:

$$i_0 = \sqrt{i_y^2 + i_z^2 + y_0^2 + z_0^2} = \sqrt{39.6^2 + 15.75^2 + 0.0^2 + 42.15^2} = 59.94\ (mm)$$

$$N_{cr,T} = \frac{1}{i_0^2}\left[GI_t + \frac{\pi^2 EI_w}{L_T^2}\right] = \frac{1}{59.94^2}\left[80{,}770 \times 896 + \frac{\pi^2 210{,}000 \times 0.0}{(1.0 \times 1000)^2}\right] = 20143\ (N)$$

$$\beta = 1 - \frac{y_0^2 + z_0^2}{i_0^2} = 1 - \frac{0.0^2 + 42.15^2}{59.94^2} = 0.5055$$

$$N_{cr,TF} = \frac{N_{cr,z}}{2\beta}\left[1 + \frac{N_{cr,T}}{N_{cr,z}} - \sqrt{\left(1 - \frac{N_{cr,T}}{N_{cr,z}}\right)^2 + 4\left(\frac{z_0}{i_0}\right)^2 \frac{N_{cr,T}}{N_{cr,z}}}\right]$$

$$= \frac{345400.8}{2 \times 0.5055}\left[1 + \frac{20143}{345400.8} - \sqrt{\left(1 - \frac{20143}{345400.8}\right)^2 + 4\left(\frac{42.15}{59.94}\right)^2 \frac{20143}{345400.8}}\right] = 19562\ (N)$$

As $N_{cr,TF} = 19562\ (N) < 20143(N) = N_{cr,T}$

$$\rightarrow \bar{\lambda}_T = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,TF}}} = \sqrt{\frac{328.4 \times 350}{19562}} = 2.424$$

For T section, the buckling curve for torsional-flexural buckling is c and $\alpha = 0.49$

$$\Phi_T = 0.5\left[1 + \alpha\left(\bar{\lambda}_T - 0.2\right) + \bar{\lambda}_T^2\right] = 0.5[1 + 0.49(2.424 - 0.2) + 2.424^2] = 3.982$$

$$\chi_T = \frac{1}{\Phi_T + \sqrt{\Phi_T^2 - \bar{\lambda}_T^2}} = \frac{1}{3.982 + \sqrt{3.982^2 - 2.424^2}} = 0.14$$

$$N_{bT,Rd} = \frac{\chi_T A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.14 \times 328.4 \times 350}{1.0} = 16092\ (N)$$

Member Flexural Capacity: the flexural capacity is calculated considering the limit states of lateral-torsional buckling, and local buckling. Distortional buckling is not considered as there is no lip stiffener.

1. Local buckling:

   The effective width method is utilized to calculate the nominal flexural strength in consideration of local and distortional buckling with the compressive stress in the top flange of $f_{yb} = 350\ (N/mm^2)$. As the section is subjected to positive moment, the top flange is under compression and it is considered an outstand unstiffened element:

$$\psi = 1$$

$$k_\sigma = 0.43$$

$$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194$$

$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{49/2}{28.4 \times 0.8194\sqrt{0.43}} = 1.6055 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{1.6055 - 0.188}{1.6055^2} = 0.55 \le 1.0$$

$$b_{eff} = \rho b_p = 0.55 \times 49 = 26.95\ (mm)$$

The T section consisting of the effective area of the top flange and gross area of the web under positive bending has the top flange in compressive stress at yield of $350\ (N/mm^2)$ and the bottom of the web in tensile strain greater than yield strain. It is permitted to consider partially plastic section modulus. Therefore, portion of the web subjected to tensile strain greater than yield strain will have tensile stress at yield $f_{yb} = 350\ (N/mm^2)$. And the stress distribution is taken as bilinear in the tension zone and linear in the compression zone as shown on the right in Figure 1:

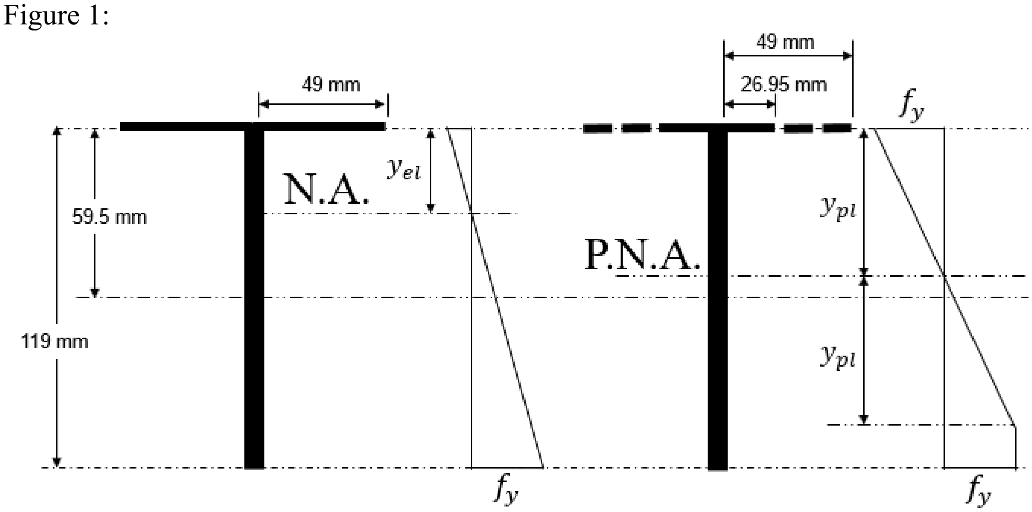


Figure 1: Sections and stress distributions – Left: gross section and linear stress distribution – Right: section with effective flange and gross web and bilinear stress distribution.

The location of plastic neutral axis y_{pl} is determined by equating the compressive and tensile forces as follows:

$$\left(2b_{eff} + 2\frac{1}{2}y_{pl}\right)tf_{yb} = \left[2\frac{1}{2}y_{pl} + 2\left(h_p - 2y_{pl}\right)\right]tf_{yb}$$

$$\rightarrow\ b_{eff} = \left(h_p - 2y_{pl}\right) \rightarrow\ y_{pl} = \frac{h_p - b_{eff}}{2} = \frac{119 - 26.95}{2} = 46.025\ (mm)$$

The web is considered an outstand unstiffened element under stress gradient.
Since the stress distribution is bilinear, the stress ratio is allowed to be taken as $\psi = -1.0$

$$k_\sigma = 1.7 - 5\psi + 17.1\psi^2 = 1.7 - 5(-1) + 17.1(-1)^2 = 23.8$$

$$\bar{\lambda}_{p,b} = \frac{h_p/(2t)}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{119/4}{28.4 \times 0.8194\sqrt{23.8}} = 0.262 < 0.748 \rightarrow\ \rho = 1.0$$

The web is fully effective.
The plastic neutral axis of the effective section is as calculated previously:
$y_{pl} = 46.025\ (mm)$

$$I_{el} = 2b_{eff}ty_{pl}{}^2 + 2\left[\frac{2ty_{pl}{}^3}{12} + 2ty_{pl}\left(\frac{y_{pl}}{2}\right)^2\right]$$

$$= 2 \times 26.95 \times 2 \times 46.025^2 + 2\left[\frac{2 \times 2 \times 46.025^3}{12} + 2 \times 2 \times 46.025\left(\frac{46.025}{2}\right)^2\right] = 488339\ (mm^4)$$

$$W_{eff,c} = \frac{I_{el}}{y_{pl}} + 2t\left(h_p - 2y_{pl}\right)\left(h_p - y_{pl} - \frac{h_p - 2y_{pl}}{2}\right)$$

$$= \frac{488339}{46.025} + 2 \times 2 \times (119 - 2 \times 46.025)\left(119 - 46.025 - \frac{119 - 2 \times 46.025}{2}\right) = 17023.4\ (mm^3)$$

$$I_y = W_{eff,c} y_{pl} = 17023.4 \times 46.025 = 783484\ (mm^4)$$

$W_{eff,t} = W_{eff,c} = 17023.4\ (mm^3)$ because the stress distribution is bilinear on the tension side and the bottom edge is also at yield stress.

$$M_{c,Rd} = \frac{W_{eff,c} f_{yb}}{\gamma_{M0}} = \frac{17023.4 \times 350}{1.0} = 5958190\ (N-mm)$$

2. Lateral-torsional buckling:

Due to the concentrated loading and simply support condition at both ends of the column:

$M_1 = -500{,}000\ N-mm$ and $M_2 = 3{,}000{,}000\ N-mm$

$$\psi = \frac{M_1}{M_2} = -\frac{500{,}000}{3{,}000{,}000} = -0.1667$$

$k_w = 1.0$ and $K_{LTB} = 1.0$

$$C_1 = (0.176\psi^2 - 0.461\psi + 0.625)(-1.338K_{LTB}^2 + 1.140K_{LTB} + 3.210)$$
$$= [0.176 \times (-0.1667)^2 - 0.461 \times (-0.1667) + 0.625](-1.338 \times 1.0^2 + 1.140 \times 1.0 + 3.210) = 2.13$$
$$C_3 = (2.01\psi^3 - 3.647\psi^2 + 2.2\psi + 7.783)(0.412K_{LTB}^2 - 0.929K_{LTB} + 0.639)$$
$$= [2.01 \times (-0.1667)^3 - 3.647 \times (-0.1667)^2 + 2.2 \times (-0.1667) + 7.783](0.412 \times 1.0^2 - 0.929 \times 1.0 + 0.639) = 0.88$$

$C_2 = 0$

The neutral axis of the gross section measured from top fiber is 42.15 (mm) → $z_a = 42.15\ (mm)$ as the load is applied on the top flange

$z_g = z_a - z_s = z_a - z_0 = 42.15 - 42.15 = 0\ (mm)$

$z_j = 47.1\ (mm)$

$L_{cr} = 1000\ (mm)$

$I_z = 166649.6\ (mm^4)$

$I_w = 0.0\ (mm^6)$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L_{cr}^2}\left\{\left[\left(\frac{K_{LTB}}{k_w}\right)\frac{I_w}{I_z} + \frac{L_{cr}^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j\right)^2\right]^{0.5} - \left(C_2 z_g - C_3 z_j\right)\right\}$$
$$= 63029336\ (N-mm)$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,c} f_{yb}}{M_{cr}}} = \sqrt{\frac{17023.4 \times 350}{63029336}} = 0.307$$

The applicable buckling curve is b and $\alpha_{LT} = 0.34$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.5[1 + 0.34(0.307 - 0.2) + 0.307^2] = 0.566$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.566 + \sqrt{0.566^2 - 0.307^2}} = 0.961 \leq 1.0$$

$$M_{b,Rd} = \chi_{LT} W_{eff,y,min} \frac{f_{yb}}{\gamma_{M1}} = 0.961 \times 17023.4 \frac{350}{1.0} = 5728005\ (N-mm)$$

Member Shear Capacity:

$$\bar{\lambda}_w = 0.346 \frac{s_w}{2t} \sqrt{\frac{f_{yb}}{E}} = 0.346 \frac{119}{2 \times 2} \sqrt{\frac{350}{210000}} = 0.42$$

$$\rightarrow \bar{\lambda}_w < 0.83$$

$$\rightarrow f_{bv} = 0.58 f_{yb} = 0.58 \times 350 = 203\ (N/mm^2)$$

$$V_{b,Rd} = \frac{h_w (2t) f_{bv}}{\gamma_{M0}} = \frac{119 \times 2 \times 2 \times 203}{1.0} = 96628\ (N)$$

Combined D/C ratio:

$$k_{yy} = k_{zz} = k_{yz} = k_{zy} = 1.0$$

By observation, the combination D/C ratio by Equation 6.36 in Eurocode 3 1-3 2006 governs the design:

$$\frac{D}{C} = \left(\frac{N_{Ed}}{N_{b,Rd}}\right)^{0.8} + \left(\frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{by,Rd}}\right)^{0.8} + \left(\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{bz,Rd}}\right)^{0.8}$$

$$= \left(\frac{1000}{16092}\right)^{0.8} + \left(\frac{3{,}000{,}000}{5728005}\right)^{0.8} + \left(\frac{0+0}{M_{bz,Rd}}\right)^{0.8} = 0.704$$